Y11 - Using Graphs (H) - KO

What do I need to be able to do?

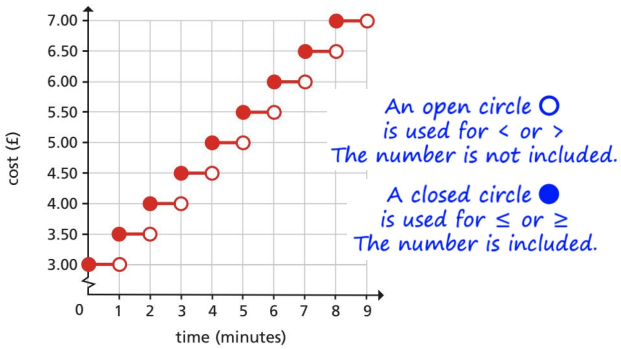
- Construct & interpret conversion graphs & other real-life straight line graphs
- Construct & interpret piecewise graphs
- Interpret & construct distance-time graphs
- Interpret velocity-time graphs to calculate speed & distance

Keywords & definitions:

- **Conversion** - change a value from one form to another
- **Approximate** - a value that is not exact, but is close enough to be used
- **Solution** - where a graph intercepts the x-axis
- **Tangent** - a straight line that touches a curve at one point only

Piecewise Graphs

The graph shows the cost of a phone call to the USA.
The cost of a call that is less than a minute long is £3

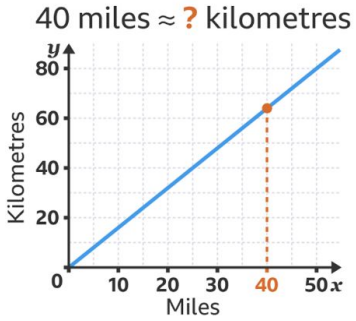


An open circle ○ is used for < or > The number is not included.

A closed circle ● is used for ≤ or ≥ The number is included.

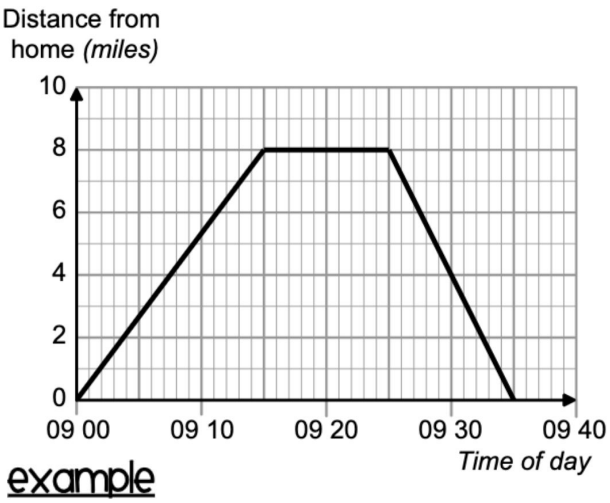
Conversion Graphs

Conversion graphs are straight line graphs that show a relationship between two units and can be used to convert from one to another. They are very useful to solve real-life problems.



distance-time graphs

A distance-time graph shows how a journey takes place, with the distance from a starting point plotted over time.



example

The graph shows Lucy's journey from home to the supermarket and back home.

a) What time did she arrive at the supermarket?

At 9.15 - when the line goes flat.

b) How long did she stay at the supermarket?

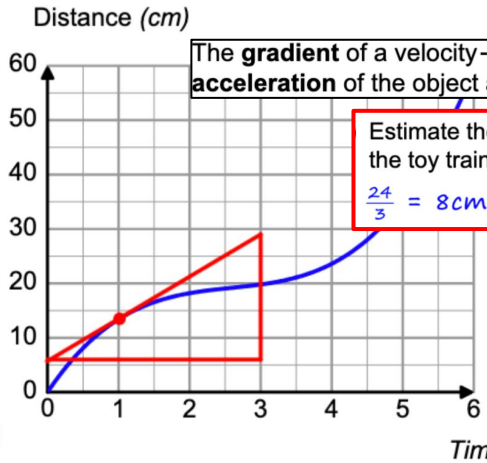
9.15 until 9.25 - 10 minutes

c) At what speed did Lucy travel home?

8 miles in 10 minutes = 48mph

velocity-time graphs

Velocity is speed in a given direction. A velocity-time graph shows the speed of an object at different times over a journey.

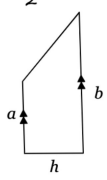


The **gradient** of a velocity-time graph tells us the **acceleration** of the object at that moment in time.

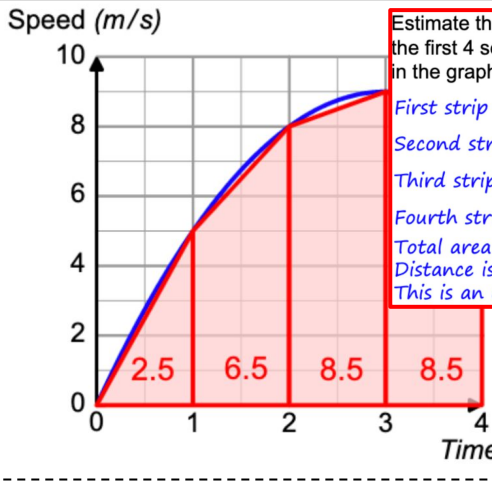
Estimate the instantaneous speed of the toy train after 1 second.

$\frac{24}{3}$ = 8cm/s

Area of Trapezium $= \frac{a+b}{2} \times h$



The **area** under a velocity-time graph tells us the **distance** the object has travelled during the time. To estimate the area, we split the area under the curve into strips of triangles and trapezia.



Estimate the distance travelled during the first 4 seconds of the journey shown in the graph. Use four strips of equal width.

First strip area = $\frac{1 \times 5}{2}$ = 2.5

Second strip area = $\frac{5+8}{2} \times 1$ = 6.5

Third strip area = $\frac{8+9}{2} \times 1$ = 8.5

Fourth strip area = $\frac{9+8}{2} \times 1$ = 8.5

Total area = 26

Distance is approximately 26m.

This is an under-estimate.

Y11 - Multiplicative Reasoning (H) - KO

What do I need to be able to do?

- Understand the fundamentals of proportional relationships and use the formulas
- Apply mathematical knowledge to concepts of money
- Work with and calculate compound measures

Keywords & definitions:

Proportion - two ratios or fractions being equal
Direct Proportion - when 2 quantities change at the same rate
Inverse Proportion - a relationship between 2 quantities where an increase in one quantity results in a proportional decrease in the other, and vice versa
Best Value - cheapest price per unit or for the same amount
Density - how tightly packed matter is. Measured as mass within a given volume
Speed - how fast something is moving. Measured as distance traveled per time unit
Pressure - measured as the force exerted on a surface per unit area

direct proportion

When two quantities maintain a constant ratio, we say they are in direct proportion. *As one increases, the other increases at the same rate*

examples

The symbol $\propto$ is known as the **proportionality symbol**.
$y \propto x$ means 'y is directly proportional to x'

If $y \propto x$, the equation $\boldsymbol{y = kx}$ can be formed, where $\boldsymbol{k}$ is the **constant of proportionality**.
The graph of this equation is a straight line through the origin with a gradient of k.

y is directly proportional to x. Complete the table and write an equation connecting x and y.

x	4		8	22
y	10	15		

↓

x	4	6	8	22
y	10	15	20	55

×2.5

Use the given pair of values to work out the constant of proportionality, k.

$y \propto x$, so $y = kx$
Given $y = 10$ when $x = 4$,
$10 = k \times 4$
$2.5 = k$
The equation is $y = 2.5x$

currency conversion

Where necessary, round currency values to 2 decimal places.

examples

£1 = $1.21
Change £250 into dollars.

£1	$1.21
£250	$____

×1.21 ↓ ×1.21

$302.50

£1 = €1.16
Change €400 into pounds.

£1	€1.16
£____	€400

÷1.16 ↓ ÷1.16

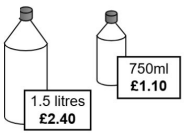
£344.83

value for money

A large bottle of lemonade (1.5 litres) costs £2.40

A small bottle of lemonade (750ml) costs £1.10

Which size bottle gives better value for money?



Large Bottles

Volume in ml	Cost (£)
1500	2.40
750	1.20

÷2

With a large bottle you are paying £1.20 for 750ml. With a small bottle 750ml only costs £1.10, so the small bottle is better value for money.

speed

examples

Ruby walks 20 metres in 4 seconds
Work out Ruby's speed.

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$
$= \frac{20}{4}$
$= 5\ m/s$

Stewart cycles at an average speed of 12 km/h
Work out how far Stewart cycles in 2 hours.

Distance = Speed × Time
$= 12 \times 2$
$= 24$ km

Tina drives at an average speed of 50 mph
How long will it take for Tina to drive 150 miles?

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$
$= \frac{150}{50}$
$= 3$ hours

converting units of speed

Change a speed of 2.88 kilometres per hour into metres per second.

2.88 kilometres = 2880 metres.

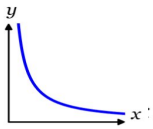
Answer = 0.8m/s

Distance	Time
2880m	1 hour
48m	1 minute
0.8m	1 second

÷ 60
÷ 60

inverse proportion

If two quantities are inversely proportional, when one of the quantities is *multiplied* by value, the other is *divided* by the same value.



examples

$y \propto \frac{1}{x}$ means 'y is inversely proportional to x'

If $y \propto \frac{1}{x}$, the equation $\boldsymbol{y = \frac{k}{x}}$ can be formed, where $\boldsymbol{k}$ is the **constant of proportionality**.
The graph of this equation is part of a reciprocal graph.

The time taken to clean all of the rooms in a hotel is inversely proportional to the number of cleaners. Work out the missing values:

Number of cleaners	8	12	
Time taken (hours)	3		6

$k = 8 \times 3 = 24$ ↓

Number of cleaners	8	12	4
Time taken (hours)	3	2	6

y is inversely proportional to x
When $x = 4$, $y = 8$.
Work out x when $y = 2$

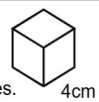
$y \propto \frac{1}{x} \rightarrow y = \frac{k}{x}$
$8 = \frac{k}{4}$
$k = 32$
Subs. $y = 2$
$y = \frac{32}{x}$
$2 = \frac{32}{x}$
$x = 16$

density

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ g/cm³ or kg/m³.

examples

A cube has side lengths of 4cm and a mass of 83g.
Work out the density of the cube corrrect to 2 significant figures.



Density = Mass ÷ Volume
$= 83 \div 64$
$= 1.296875$
$= 1.3\ g/cm^3$ (2 s.f.)

A silver coin has a mass of 6.5g and density 10.5g/cm³.
Work out the volume of the coin to 3 significant figures.

$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \rightarrow \text{Volume} = \frac{\text{Mass}}{\text{Density}}$
$\text{Volume} = \frac{6.5}{10.5} = 0.619 cm^3$

combining densities

Liquid A has a density of 1.06 g/cm³.
Liquid B has a density of 0.84 g/cm³.

2 litres of liquid A is mixed with 1 litre of liquid B to make liquid C.

Work out the density of liquid C.

	Density	Mass	Volume
A	1.06g/cm³	2120g	2000cm³
B	0.84 g/cm³	840g	1000cm³
C	0.99g/cm³	2960g	3000cm³

Fill in the table with all the information given, and work out any other values possible.

Recall that 1 litre = 1000cm³

The final density is the total mass divided by the total volume.

pressure

$\text{pressure} = \frac{\text{force}}{\text{area}}$. units are N/m² or N/cm²

A wooden toy box is resting on a counter.
The box covers an area of 0.4m²
The weight of the box exerts a force of 30 Newtons on the counter.
Work out the pressure on the counter created by the box.

$P = \frac{30}{0.4}$
$= 75\ N/m^2$

Retrieval Quiz

Answers