

Properties of Number

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors

Conjecture: a statement that might be true (based on reasoning) but is not proven

Counterexample: a special type of example that disproves a statement

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Divisibility Rules

A number is divisible by...

2	if the last digit is even or 0
3	if the sum of the digits is divisible by 3
4	if the last two digits are divisible by 4
5	if the last digit is 0 or 5
6	if the number is divisible by both 2 and 3
9	if the sum of the digits is divisible by 9
10	if the last digit is 0

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

Factors

Arrays can help represent factors

5×2 or 2×5

Factors of 10
1, 2, 5, 10

10×1 or 1×10

The number itself is always a factor

Factors and expressions

$x \ x \ x \ x \ x \ x$

$6x \times 1$ OR $6 \times x$

Factors of $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$x \ x$

$x \ x$

$x \ x \ x$

$x \ x \ x$

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

odd

even

odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful — an extra counter is added to each new row

Odd two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

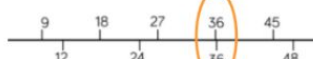
LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match



Comparing fractions

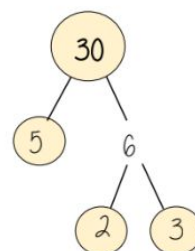
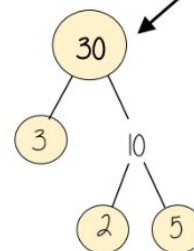
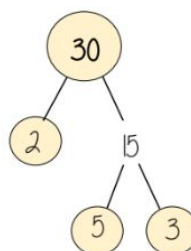
Compare fractions using a LCM denominator

$\frac{3}{5}$ and $\frac{7}{10}$

$\frac{6}{10}$ and $\frac{7}{10}$

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g. 60: 30×2 or $2 \times 3 \times 5 \times 2$

150: 30×5 or $2 \times 3 \times 5 \times 5$

YEAR 8 - PROPORTIONAL REASONING...

Ratio and Scale

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved

Keywords

Ratio: a statement of how two numbers compare

Equal Parts: all parts in the same proportion, or a whole shared equally

Proportion: a statement that links two ratios

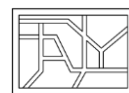
Order: to place a number in a determined sequence

Part: a section of a whole

Equivalent: of equal value

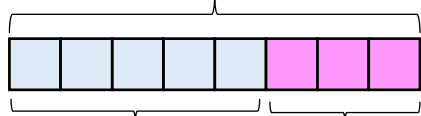
Factors: integers that multiply together to get the original value

Scale: the comparison of something drawn to its actual size.



Representing a ratio

This is the "whole" — boys and girls together



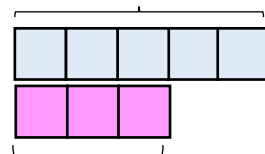
This represents the 5 boys

This represents the 3 girls

"For every 5 boys there are 3 girls"

5:3

This represents the 5 boys



This represents the 3 girls

Double Number Line

This is the "whole" — boys and girls together

Order is Important

"For every dog there are 2 cats"



Dogs: Cats



1:2

The ratio has to be written in the same order as the information is given

e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

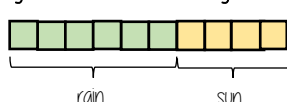
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

+ by 2 ↓

3:2



rain

sun



Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiplies into them is 2)

"For every 3 days of rain there are 2 days of sun" — when this happens twice the ratio becomes 6:4.

Ratio In (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit.

Therefore Divide by 4

4 : 20
1 : 5

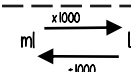
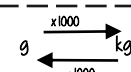
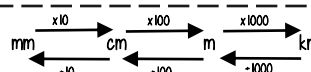
This side has to be divided by 4 too — to keep in proportion

*The n part does not have to be an integer for this type of question

Units are important:

When using a ratio — all parts should be in the same units

Useful Conversions



Sharing a whole into a given ratio

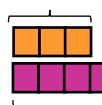
James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3:4

James



Lucy

£350 ÷ 7 = £50

□ = one part = £50

Find the value of one part

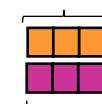
Whole: £350
7 parts to share between
(3 James, 4 Lucy)

Put back into the question

James: Lucy

(x 50) 3:4 (x 50)
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue: Red

5:1

□ = one part = 10 pens

Blue pens



Red pens

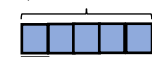
One unit = 10 pens

Put back into the question

Blue: Red

(x 10) 5:1 (x 10)
50:10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

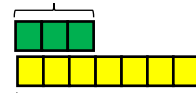


Ratio as a fraction

Trees: Flowers

3:7

Trees



Flowers

Fraction of trees

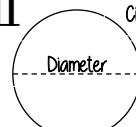
There are 3 parts for trees

Number of parts of in group
Total number of parts

3
10

Tree parts 3 + Flower parts 7 = 10

π



Circumference

Diameter

The ratio of a circles circumference to its diameter

YEAR 8 - ALGEBRAIC TECHNIQUES...

Sequences

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the n th term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order

Term: a single number or variable

Position: the place something is located

Linear: the difference between terms increases or decreases (+ or -) by a constant value each time

Non-linear: the difference between terms increases or decreases in different amounts, or by $\times$ or $\div$

Difference: the gap between two terms

Arithmetic: a sequence where the difference between the terms is constant

Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

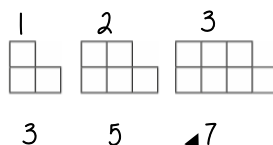
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms



Sequence in a table and graphically

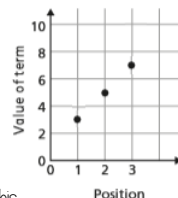
Position: the place in the sequence



"The term in position 3 has 7 squares"

Term: the number or variable (the number of squares in each image)

Graphically



In a table

Position	1	2	3
Term	3	5	7

+2 +2

Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$$3n + 7$$

$$3n^2 + 7$$

This will be linear - note the single power of n . The values increase at a constant rate

This is not linear as there is a power for n

$$2n - 5 \rightarrow$$

Substitute the number of the term you are looking for in place of 'n'

eg

$$1^{\text{st}} \text{ term} = 2(1) - 5 = -3$$

$$2^{\text{nd}} \text{ term} = 2(2) - 5 = -1$$

$$100^{\text{th}} \text{ term} = 2(100) - 5 = 195$$

Checking for a term in a sequence

Form an equation

Is 201 in the sequence $3n - 4$?

$$3n - 4 = 201$$

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$$2n^2$$

2 times whatever n squared is

eg

$$1^{\text{st}} \text{ term} = 2 \times 1^2 = 2$$

$$2^{\text{st}} \text{ term} = 2 \times 2^2 = 8$$

$$100^{\text{th}} \text{ term} = 2 \times 100^2 = 20000$$

$$(2n)^2$$

2 times n then square the answer

eg

$$1^{\text{st}} \text{ term} = (2 \times 1)^2 = 4$$

$$2^{\text{st}} \text{ term} = (2 \times 2)^2 = 16$$

$$100^{\text{th}} \text{ term} = (2 \times 100)^2 = 40000$$

$$n(n + 5)$$

eg

$$1^{\text{st}} \text{ term} = 1(1 + 5) = 6$$

$$2^{\text{st}} \text{ term} = 2(2 + 5) = 14$$

$$100^{\text{th}} \text{ term} = 100(100 + 5) = 10500$$

You don't need to expand the expression

H Finding the algebraic rule

This is the 4 times table $\rightarrow 4, 8, 12, 16, 20, \dots$

$$4n$$

$$7, 11, 15, 19, 22$$

This has the same constant difference – but is 3 more than the original sequence

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence