Y10 - Working with Circles (H) - KO

What do I need to be able to do?:

- *Recognise and label parts of a circle*
- *Calculate the length of an arc*
- *Calculate the area of a sector*
- *Understand and use volume of a cone, cylinder and sphere.*
- *Understand and use surface area of a cone, cylinder and sphere.*
- *Work with similar shapes*

Keywords & definitions:

Circumference: the length around the outside of the circle – the perimeter

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

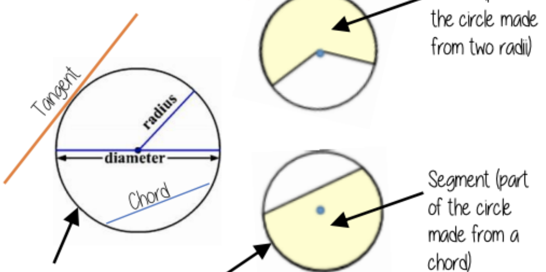
Tangent: a straight line that touches the circumference of a circle

Chord: a line segment connecting two points on the curve

Surface area: the total area of the surface of a 3D shape.

Parts of a circle

R



Sector (part of the circle made from two radii)

Segment (part of the circle made from a chord)

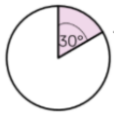
Circumference

An arc is a part of the circumference

Fractional parts of a circle

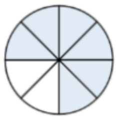
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

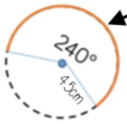
$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length



Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$

Arc length = $\frac{\theta}{360}$ ×circumference

$= \frac{240}{360} \times 9\pi$

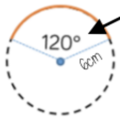
$= \frac{2}{3} \times 9\pi$ $= \underline{6\pi}$

Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ ×circumference $+ 2r$ $= \underline{6\pi + 9}$

Sector area



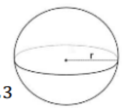
Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$

Sector area = $\frac{\theta}{360}$ × area of circle

$= \frac{120}{360} \times 36\pi$

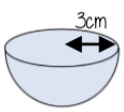
$= \frac{1}{3} \times 36\pi$ $= \underline{12\pi}$

Volume of a sphere



Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value



Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

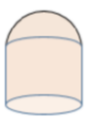
$= \frac{4}{3} \times \pi \times 27$ $= \underline{36\pi}$

A hemisphere is half the volume of the overall sphere

$= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g cones and cylinders



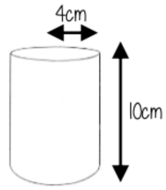
Volume of a cone and a cylinder

Volume Cylinder = $\pi r^2 h$

A cylinder is a prism – cross section is a circle

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base



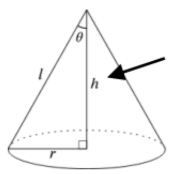
$V = \pi r^2 h$

$= \pi \times 4^2 \times 10$

$= \pi \times 160$

$= \underline{160\pi}$ cm²

Give your answer in terms of π' means NOT in terms of pi $= \underline{502.7}$cm²

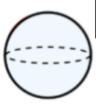


The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Surface area of a sphere

Surface area = $4\pi r^2$



Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

$= \underline{100\pi}$

The curved surface area of a sphere

A hemisphere has the curved surface AND a flat circular face

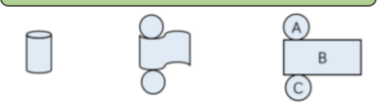
$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= \underline{75\pi}$

Surface area of cones and cylinders

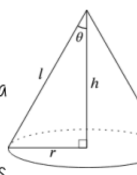
Surface area cylinder = $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles

Curved surface area Cone = πrl



Look out for the use of Pythagoras to calculate the length l

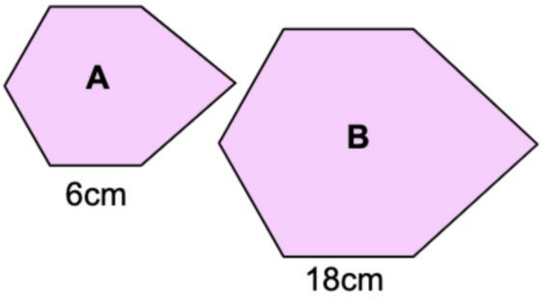
Total surface area = curved face + circle face (area of base)

area and surface area of enlarged shapes

When a 2D shape is enlarged,
the area scale factor = length scale factor^2.
For example, if the lengths increase by a factor of 3, the area will get 9 times larger.

example

The area of shape A is $50cm^2$.
What is the area of shape B?



Length scale factor
= 6 : 18
= 1 : 3

Area scale factor
= 1 : 9
= 50 : $450cm^2$

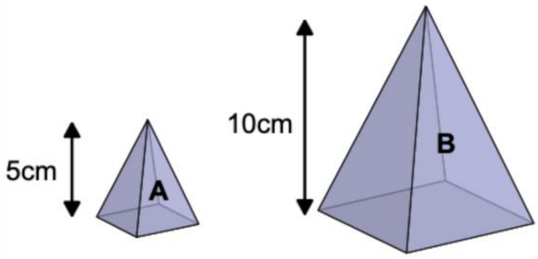
Area = **450**cm^2

volume of enlarged shapes

When a 3D shape is enlarged,
the volume scale factor = length scale factor^3.
For example, if the lengths increase by a factor of 3, the volume will get 27 times larger.

example

The volume of shape A is $12cm^3$.
What is the volume of shape B?



Length scale factor
= 5 : 10
= 1 : 2

Volume scale factor
= 1 : 8
= 12 : $96cm^3$

Volume = **96** cm^3

Retrieval Quiz

Answers

Constructions (H)

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles

Keywords

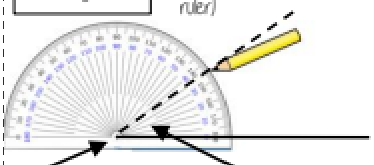
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

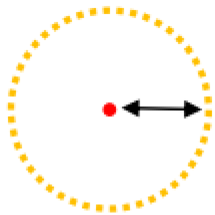
Make a mark at 35° with a pencil And join to the angle point (use a ruler)



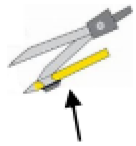
The angle

Make sure the cross is at the end of the line (where you want the angle)

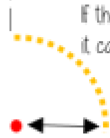
Locus of a distance from a point



All points are equidistant (the same distance) from the fixed point in the middle

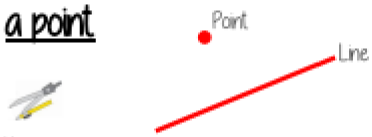


If the point is in the corner it can only make a quarter circle

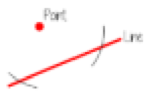


Equipment needed
The radius is the distance from the fixed point

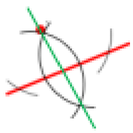
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



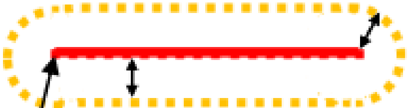
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from a straight line

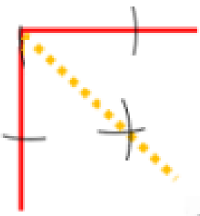


All points are equidistant (the same distance) from line

The ends of the line are fixed points

Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

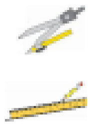
Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

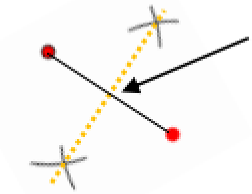
From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle



Join the vertex to the intersection

Locus equidistant from two points



Also a perpendicular bisector

Because if the points are joined, this new line intersects it at a 90°

Join the intersections with a ruler.
All points on this line are equidistant from both points

Keep the compass the same size and draw two arcs from each point

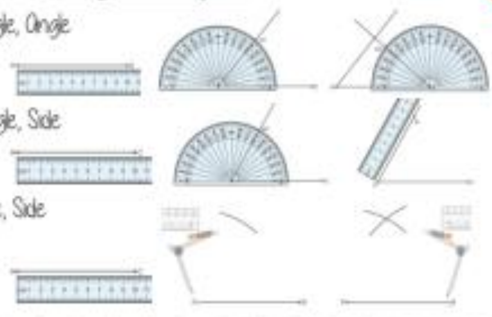
Constructing Triangles

Link to steps → R

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



Congruence similarity and enlargement (H)

What do I need to be able to do?

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords & definitions:

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Centre of enlargement: the point the shape is enlarged from

Similar: when one shape can become another with a reflection, rotation, enlargement or translation.

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar situations

Parallel: straight lines that never meet (equal gradients)

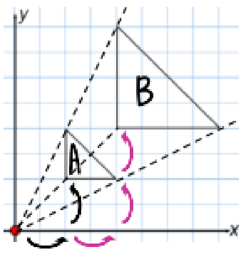
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

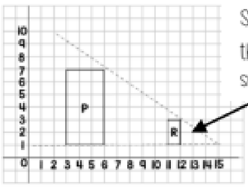
The distance from the point enlarges by 2



Fractional scale factors R

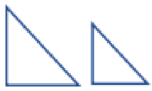
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15, 1)



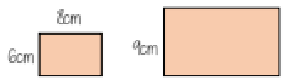
SF $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change
eg if a triangle gets bigger the angles can not go above 180°

Similar shapes

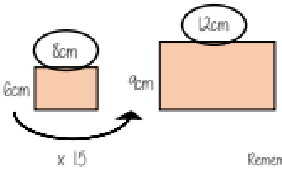


Scale Factor
Both sides on the bigger shape are 1.5 times bigger

Compare sides 6 : 9 **2 : 3** 8 : 12 **2 : 3**

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

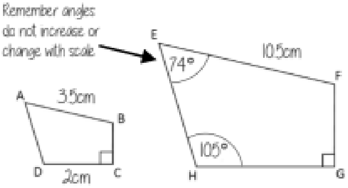
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

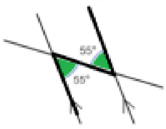
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines R

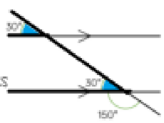
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

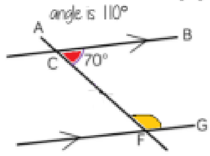
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

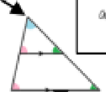
Because co-interior angles have a sum of 180° the highlighted angle is 110°



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Similar triangles

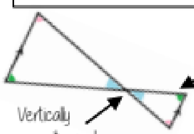
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

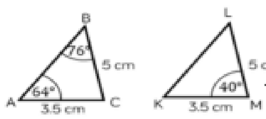


All the angles in both triangles are the same and so similar

Vertically opposite angles

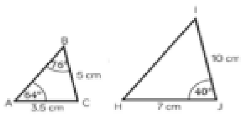
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



ACB = KML

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

Retrieval Quiz

Answers