Pythagoras and Trigonometry (H)

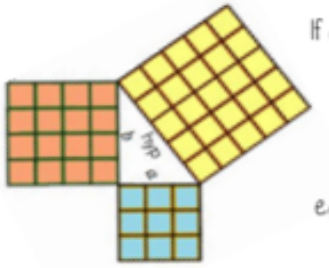
What do I need to be able to do?

- Determine whether a triangle is right angled
- calculate missing sides & angles in right-angled and non right-angled triangles
- Select the appropriate method to find missing sides and angles in right angled and non right-angled triangles
- Work with key angles in right-angled triangles (non-calculator trigonometry including exact values)
- Use Pythagoras and trigonometry in 3D shapes
- Use the formula 1/2abSinC to find the area of a triangle

Keywords & definitions:

Hypotenuse: the largest side on a right-angled triangle. Always opposite the right angle. It is also defined as the longest side of a right-angled triangle, and it is the side opposite the right-angle.
Opposite: the side opposite the angle of interest.
Adjacent: the side next to the angle of interest.
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

eg $a^2+b^2 = hypotenuse^2$

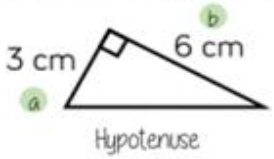
$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Pythagoras Theorem

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$3^2+6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

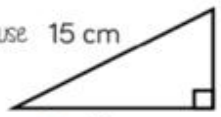
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides

$\sqrt{45} = \text{hypotenuse}$

$6.71cm = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2+b^2 = 15^2$

1. Substitute in the values you are given

$144 + b^2 = 225$

$-144 \quad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

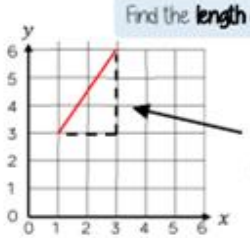
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the **hypotenuse**

$$a^2 + b^2 = \text{hypotenuse}^2$$

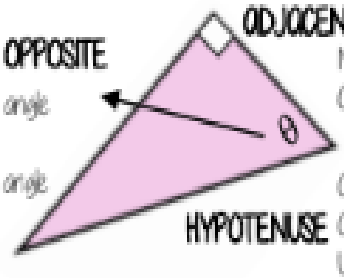
The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes

Right angled trigonometry

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

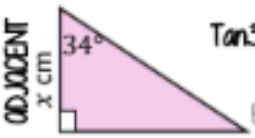
SOH CAH TOA

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj.}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Tangent ratio: side lengths

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



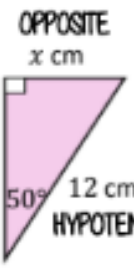
$\tan 34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \tan 34 = 10$

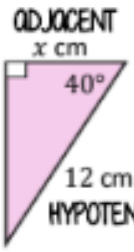
$x = \frac{10}{\tan 34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio



$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Sin, Cos, Tan: Angles

Inverse trigonometric functions

Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\frac{3}{4}$

$\theta = 36.9°$.

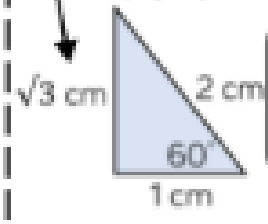
$\theta = \tan^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \sin^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \cos^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

This side could be calculated using Pythagoras

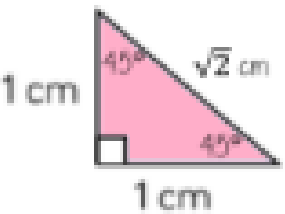


Because trig ratios remain the same for similar shapes you can generalise from the following statements

$\tan 30 = \frac{1}{\sqrt{3}}$ $\tan 60 = \sqrt{3}$

$\cos 30 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$

$\sin 30 = \frac{1}{2}$ $\sin 60 = \frac{\sqrt{3}}{2}$



$\tan 45 = 1$

$\cos 45 = \frac{1}{\sqrt{2}}$

$\sin 45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\tan 0 = 0$

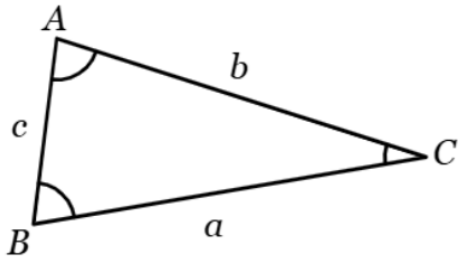
~~$\tan 90$~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle

$\sin 0 = 0$ $\sin 90 = 1$

$\cos 0 = 1$ $\cos 90 = 0$

To find missing lengths and angles in any triangle, including those without a right angle, the sine rule can be used: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



The capital letters refers to the angles. Angle A is opposite side a.

If we wish to find an angle, the reciprocal formula can be used: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

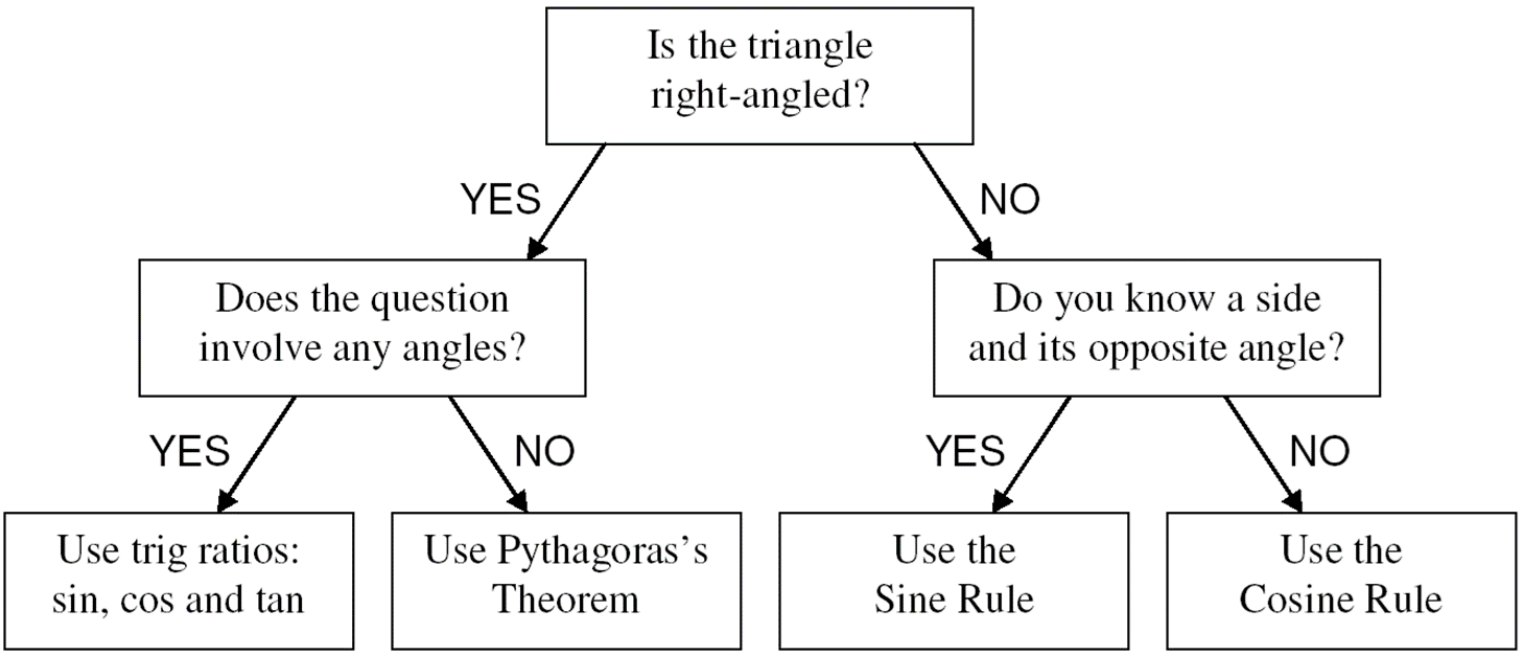
The **sine rule** is useful when we have two pairs of connected sides and angles, with one unknown.
The **cosine rule** is useful when we are working with a single angle and all three sides, with one unknown.

The cosine rule is: $\boldsymbol{a^2 = b^2 + c^2 - 2bc(\cos A)}$

or to find an angle: $\boldsymbol{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$

Area of a triangle = $\boldsymbol{\frac{1}{2}ab\sin C}$

Non right angled trigonometry



Representing solutions of equations and inequalities (H)

What do I need to be able to do?

- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords & definitions:

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: An equation where both sides have variables that cause the same answer includes ≡

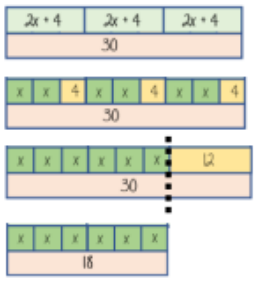
Linear: an equation or function that is the equation of a straight line

Intersection: the point that two lines meet

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations (R)

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$-6 \quad -6$

$x = 3$

Substitute to check your answer. This could be negative or a fraction or decimal

Form and solve inequalities (R)

Two more than treble my number is greater than 11

Form

$x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$3x + 2 > 11$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

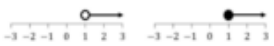
$x > 3$

Solutions on a number line

$x < 1$ $\quad x \leq 1$

Both represent values less than 1

Includes the value 1



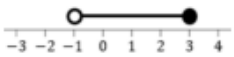
$x > 1$ $\quad x \geq 1$

Both represent values more than 1

Includes the value 1

- Includes the value it sits above
- Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



$-1 < x \leq 3$

This includes the integer values 0,1,2,3

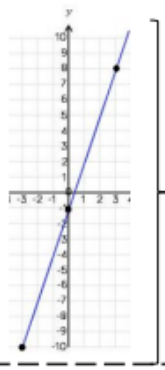
Plotting straight line graphs (R)

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

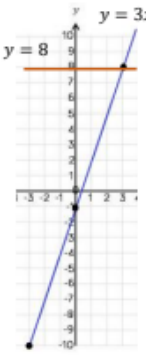
Remember to join the points to make a line

Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



$3x - 1 = 8$

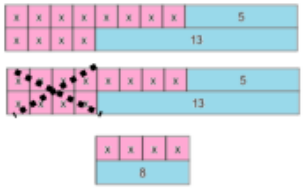
Remember equation of a line format is $y = mx + c$

The solution is the point the two lines meet

(3,8)

Equations: unknown on both sides (R)

$8x + 5 = 4x + 13$



$8x + 5 = 4x + 13$

$-4x \quad -4x$

$4x + 5 = 13$

$-5 \quad -5$

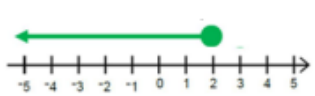
$4x = 8$

$\div 4 \quad \div 4$

$x = 2$

Inequalities: unknown on both sides

$8x + 5 \leq 4x + 13$ → $x \leq 2$



Any value 2 or less will satisfy this inequality

Retrieval Quiz

Answers