YEAR 8 - PROPORTIONAL REASONING...

Multiplying and Dividing Fractions

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts.

Whole: a positive number including zero without any decimal or fractional parts.

Commutative: an operation is commutative if changing the order does not change the result.

Unit Fraction: a fraction where the numerator is one and denominator a positive integer.

Non-unit Fraction: a fraction where the numerator is larger than one.

Dividend: the amount you want to divide up.

Divisor: the number that divides another number.

Quotient: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

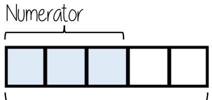
Reciprocal: a pair of numbers that multiply together to give 1.

Representing a fraction

$\frac{\text{Numerator}}{\text{Denominator}}$

$\frac{3}{5}$

Number of parts represented
Numerator



Number of parts to make up the whole
Denominator

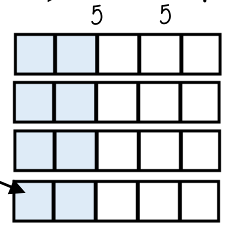
ALL PARTS of a fraction are of equal size

Repeated addition = multiplication by an integer

$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

Integer (Whole number)

Each part represents $\frac{1}{5}$

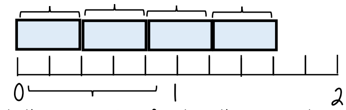


How many parts are shaded?
What each part represents

Revisit
When adding fractions with the same denominator = add the numerators

$= \frac{8}{5} \leftarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

$= 1\frac{3}{5}$



Each whole is split into the same number of parts as the denominator

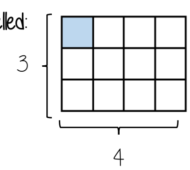
Multiplying unit fractions

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Multiplying non-unit fractions

Shade in 3 parts

Repeat it on this many rows

$\frac{3}{4} \times \frac{2}{3}$

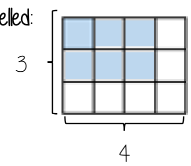
This many columns

This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Modelled:



Total number of parts in the diagram

Quick Multiplying and Cancelling down

$\frac{\cancel{3}^{\,1}}{5} \times \frac{4}{\cancel{9}_{\,3}}$

The 3 and the 9 have a common factor and can be simplified

Quick Solving
Multiply the numerators
Multiply the denominators

$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is $\frac{1}{3}$ and vice versa

Reciprocals for division

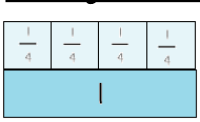
e.g.

$5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

Dividing an integer by an unit fraction



$\underbrace{1 \div \frac{1}{4}}$ $= 4$

How many quarters are in 1?

"There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes"

$5 \div \frac{1}{4} = 20$

Dividing any fractions

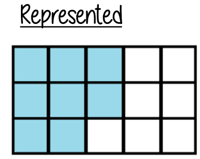
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 8 - PROPORTIONAL REASONING...

Multiplicative Change

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What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

Keywords

Proportion: a statement that links two ratios

Variable: a part that the value can be changed

Axes: horizontal and vertical lines that a graph is plotted around

Approximation: an estimate for a value

Scale Factor: the multiple that increases/ decreases a shape in size

Currency: the system of money used in a particular country

Conversion: the process of changing one variable to another

Scale: the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

x 0.5: 4 cans of pop = £2.40 → 2 cans of pop = £1.20 (x 0.5)

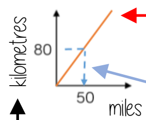
This multiplier is the same In the same way that this would be for ratio

x 3: 4 cans of pop = £2.40 → 12 cans of pop = £7.20 (x 3)

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

Conversion between currencies

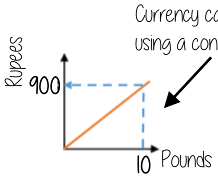
£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

x 10: £1 = 90 Rupees → £10 = 900 Rupees (x 10)

Currency can be converted using a conversion graph

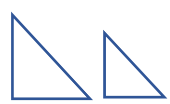


Convert 630 Rupees into Pounds

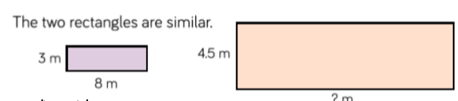
630 ÷ 90 = 7

x 7: £1 = 90 Rupees → £7 = 630 Rupees (x 7)

Ratio between similar shapes



Angles in similar shapes do not change.
e.g. if a triangle gets bigger the angles can not go above 180^0



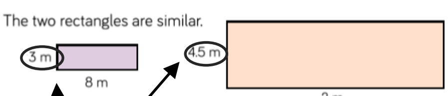
Corresponding sides

÷ 3: 3m : 4.5m → 1 m : 1.5 m (÷ 3)

x 8: 8m : 12m ← 1 m : 1.5 m (x 8)

Note
Simplify to the same ratio

Understand Scale Factor



3 x 1.5 = 4.5

This is a multiplicative change

Missing length
8 x 1.5 = 12m

Use **corresponding** sides to calculate a scale factor

Scale factor can also be calculated by:

$$\frac{\text{Bigger corresponding side}}{\text{Smaller corresponding side}}$$

Small corresponding side → x SF → Big corresponding side

Big corresponding side → ÷ SF → Small corresponding side

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life
x 10: 1cm : 30cm → 10cm : 300cm (x 10)

The car in real life is 210cm

Image : Real life
x 7: 1cm : 30cm → 7cm : 210 cm (x 7)

Interpret maps with scale factors

mm → ÷ 10 → cm → ÷ 100 → m → ÷ 1000 → km

km → x 1000 → m → x 100 → cm → x 10 → mm

1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

250 x 100 = 25000

1 cm : 25000cm

For every 1cm on my map is 25000cm in real life.

Indices & Standard Form

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Addition/ Subtraction with indices

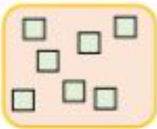
Coefficient → $5x^2 + 4x^4$ ← Power

$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}}$ →

Each square represents x^2 and each cube represents x^4

Only similar terms can be simplified. If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →



→ $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →

→ $2x^2 + 7x^4$

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Divide expressions with indices

$\frac{24}{36}$ → $\frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}}$ → $\frac{2}{3}$

$\frac{5a^3b^2}{15ab^6}$ → $\frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}$ → $\frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Addition/ Subtraction laws for indices

$3^5 \times 3^2$ → 3^7

$= \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)}$

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2$ → 3^3

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}}$ → $\frac{3^3}{3^0}$ → $\frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = \underbrace{2^3 \times 2^3 \times 2^3 \times 2^3}$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = \underbrace{2x^3 \times 2x^3 \times 2x^3 \times 2x^3}$$

The addition law applies ONLY to the powers
The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Positive powers of 10

1 billion – 1 000 000 000

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

= 32000

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Negative powers of 10

	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0.001					
$1 \times \frac{1}{1000}$	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1×10^{-3}	0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054

$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	**240**	**1**	**0.13**

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ **Not in Standard Form**

$= 6.4 \times 10^2 \times 10^3$ **Use addition for indices rule**

$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$

$= 24 \times 10^5$ **Not in Standard Form**

$= 2.4 \times 10^1 \times 10^5$ **Use addition for indices rule**

$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ **Divide the values**

$= (2 \div 4) \times 10^3$

$= 0.5 \times 10^3$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$$6 \times 10^5 + 8 \times 10^5$$

Method 1

$= 600000 + 800000$

$= 1400000$

$= 1.4 \times 10^6$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$

This is not the final answer

$= 1.4 \times 10^1 \times 10^5$

$= 1.4 \times 10^6$

Only works if the powers are the same

Multiplication and division

$$\frac{1.5 \times 10^5}{0.3 \times 10^3}$$

Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for **A** and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices	Subtraction law for indices
$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$

Using a calculator

$1.4 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 1.4 and press **x10ˣ** Then press 5 (for the power)
Press **×**
Input 3.9 and press **x10ˣ** Then press 3 (for the power)
Press **=**

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press **SHIFT** **SETUP** and then press 7 for sci mode

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8